THEORETICAL ASPECTS OF MICROSTRIP WAVEGUIDES

(Abstract)

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The Microstrip[1] is a modification of the wire-above-ground system of transmission — the wire is replaced by a narrow strip printed on a dielectric sheet which is backed by a conductor (Fig. 1).

An exact theory for such a structure is difficult because the cross section is not homogeneous and has a geometry which does not lead to separation of variables. Furthermore it is an "open" structure where the energy is not confined to a finite region as it would be in conventional or "closed" waveguides.

The Microstrip is a uniform waveguide, that is, all its transverse sections are equal. It follows that the general solution for the field, at a given frequency, is a superposition of simpler solutions, or modes, which have a z-dependance of the form exp (-jhz). The acceptable values of h form the "spectrum" just as the natural frequencies of a cavity form its frequency spectrum. In close waveguides these values of h form a discrete sequence. When there are no losses some are real and others are pure imaginary, corresponding respectively to propagating and attenuated modes.

For "open" waveguides the situation is different: the place of propagating modes is taken by surface waves, also called free modes, for which h is real and larger than the propagation constant in the medium that extends to infinity. (For this reason, these modes are also called slow waves.) The complement of the spectrum, necessary for completeness, is continuous.

Some well known illustrations of this are the cylindrical dielectric rod, the dielectric slab backed by a conductor, the cylindrical wire coated with dielectric. A similar situation occurs also in quantum mechanics for the motion of an electron near a nucleus: the free modes correspond to "bound states" and the continuous spectrum to "free electrons." The problem of propagation along a cylinder having a given surface reactance has been treated theoretically in some detail.[2] It shows how the continuous spectrum corresponds to power radiated and also that the relative importance of this radiation can only be evaluated with respect to a certain way of exciting the structure. In contrast with the close waveguide where the attenuated modes become negligible in less than a wavelength, the continuous spectrum for an open waveguide can in some cases be felt 20 or 30 wavelengths away and is detected, for instance, by attenuation measurements as a function of distance. With microstrip these measurements, for reasonable means of excitation, have shown that the perturbed region extends only a few wavelengths from the feed point.

[1] D. D. Grieg and H. F. Engelmann, "Microstrip - a new transmission technique for the kilomegacycle range," Proc. I.R.E., vol. 40, pp.1644-1659, December, 1952.

[2] T. E. Roberts, Jr., "Theory of the single-wire transmission line," Jour. Appl. Physics, vol. 24, pp.57-67, January, 1953.

The next problem would be to find the field configuration and propagation constant for the free modes. Letting u and v be the longitudinal components of electric and magnetic field, they should satisfy the equations:

$$\nabla_t^2 + k^2 - h^2 = 0 \text{ in region I, and}$$

$$\nabla_t^2 + \varepsilon k^2 - h^2 = 0 \text{ in region II.}$$

At infinity u and v must decrease in the proper way. On metal boundaries they must satisfy: $u = v_x = 0$. Across the boundary between I and II, u, v and the transverse fields

$$\frac{u_y + \zeta v_x}{k^2 - h^2} \quad \text{and} \quad \frac{v_y - \eta u_x}{k^2 - h^2}$$

should be continuous. From this last condition follows[3] that neither u nor v can be zero: there are no TE nor TM solutions.

Another approach is to consider the problem of diffraction between regions I and II. Both regions are relatively simple (known Green functions) and a pair of integral equations can be set up for the functions u and v on the boundary. They are however, coupled, which expresses in another way that there can be no pure TE or TM mode.

The existence of at least one surface mode which results from the perturbation by the dielectric of the TEM mode for the strip above ground, can be assumed. Other modes will appear according to the "size" of the cross-section. Their relative importance will depend on the method of excitation and they would be observed by measurement of the input reflection coefficient when a load is displaced along the line.

When the width of the strip and its spacing to the ground plane are small with respect to the wavelength only one mode seems to be present. Some approximations can then be made to obtain numerical results.

a) In the region near the strip the field is practically transverse. This can be ascertained from Maxwell equations on the symmetry axis ox.

b) At a large distance from this region, the field will vary with direction and distance in a simple way. This for the same reason that the pattern of a small antenna cannot contain high space harmonics.

A theory based on TEM propagation, which assumes only one dielectric[4] can be considered as giving a qualitative picture of the larger components of the field. It does explain the lack of dispersion and can be refined to give a better pre-

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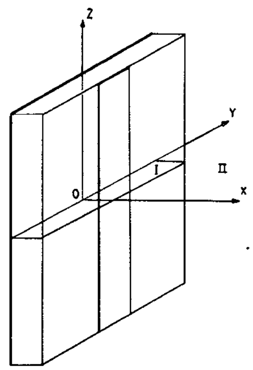
[3] L. Pincherele, "Electromagnetic waves in metal tubes filled longitudinally with two dielectrics," Phys. Rev., vol. 66, pp.118-130, September 1-15, 1944.

[4] F. Assadourian and E. Rimai, "Simplified theory of microstrip transmission systems," Proc. I.R.E., vol. 40, pp.1651-1657, December, 1952.

diction of the wavelength: following a variational procedure, Chambers[5] and Bradshaw[6] for instance, one of Maxwell's equations can be replaced by the condition that $\int(\mu|H|^2 - \varepsilon|E|^2)\,dv$ be minimum. Starting from a transverse electric field obtained by impressing the variation $\exp(-jhz)$ on the static electric field, this condition determines the value of h. It is the same as for a single dielectric having a constant equal to the weighted average of ε_I and ε_{II} with weights proportional to the energy flows in the region I and II. These weights were evaluated by making a conformal mapping of the cross-section where energy flows become proportional to the surfaces and then using graphical integration.

Because of the nature of the problem most results of importance (wavelength, attenuation) are obtained faster from measurements than from the computations which would result from the methods outlined. However, the present theoretical considerations, incomplete as they are, have been a guide in suggesting experiments and drawing conclusions from them.



$$\varepsilon = \frac{\varepsilon_I}{\varepsilon_{II}}$$

$$k = \frac{2\pi}{\lambda}$$

Fig. 1

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5 Lt. G. Chambers, "Compilation of the propagation constants of an inhomogeneously-filled waveguide," Brit. Jour. Appl. Phys., vol. 3, pp.19-21, January, 1952.

6 J. A. Bradshaw, "Calculation of the propagation constants of an inhomogeneously-filled waveguide," Brit. Jour. Appl. Phys., vol. 3, pp.332-333, October, 1952.